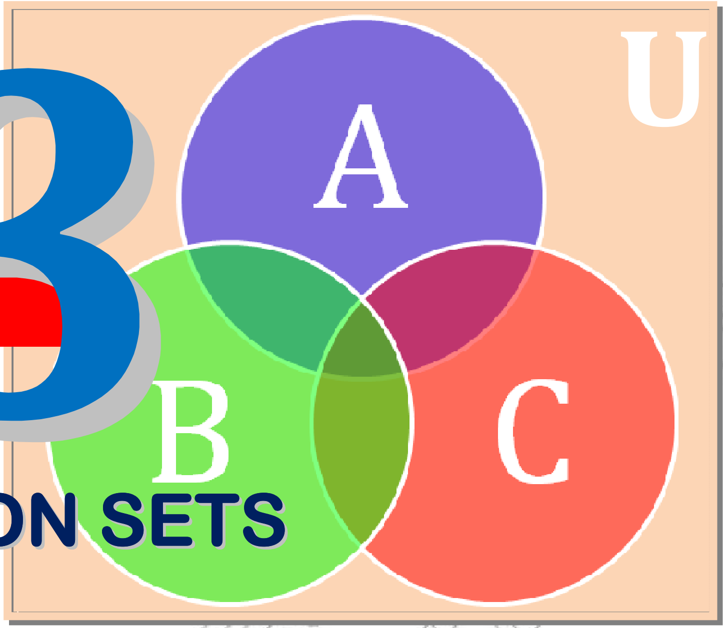
Unit 3



FURTHER ON SETS

Unit Outcomes:

After completing this unit, you should be able to:

- *understand additional facts and principles about sets.*
- *apply rules of operations on sets and find the result.*
- *demonstrate correct usage of Venn diagrams in set operations.*
- *apply rules and principles of set theory to practical situations.*

Main Contents

3.1 Ways to describe sets

3.2 The notion of sets

3.3 Operations on sets

Key Terms

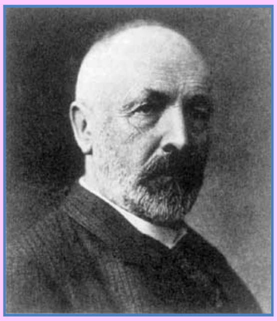
Summary

Review Exercises

INTRODUCTION

IN THE PRESENT UNIT, YOU WILL LEARN MORE ABOUT SETS. PARTICULARLY, YOU WILL DISCUSS THE DIFFERENT WAYS TO DESCRIBE SETS AND THEIR REPRESENTATION THROUGH VENN DIAGRAMS. ALSO, YOU WILL DISCUSS SOME OPERATIONS THAT, WHEN PERFORMED ON TWO OR MORE SETS, GIVE RISE TO AN SET. FINALLY, YOU WILL GO THROUGH SOME PRACTICAL PROBLEMS RELATED AND TRY TO SOLVE THEM, USING THE UNION AND INTERSECTION OF SETS.

HISTORICAL NOTE:

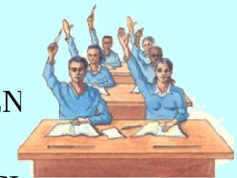


George Cantor (1845-1918)

During the latter part of the 19th century, while working with mathematical entities called infinite series, George Cantor found it helpful to borrow a word from common usage to describe a mathematical idea. The word he borrowed was set. Born in Russia, Cantor moved to Germany at the age of 11 and lived there for the rest of his life. He is known today as the originator of set theory.

3.1 WAYS TO DESCRIBE SETS

ACTIVITY 3.1



1. WHAT IS A SET? WHAT DO WE MEAN WHEN WE SAY AN ELEMENT BELONGS TO A SET?
2. GIVE TWO MEMBERS OR ELEMENTS THAT BELONG TO EACH OF THE FOLLOWING SETS
   - A THE SET OF COMPOSITE NUMBERS LESS THAN 10.
   - B THE SET OF NATURAL NUMBERS LESS THAN 50 AND DIVISIBLE BY 5.
   - C THE SET OF WHOLE NUMBERS BETWEEN.
   - D THE SET OF REAL NUMBERS BETWEEN.
   - E THE SET OF NEGATIVE INTEGERS.
   - F THE SET OF INTEGERS THAT SATISFY $(x - 2)(2x + 1) = 2x^2 - 3x - 2$.
3. A DESCRIBE EACH OF THE SETS IN QUESTION 2 BY ANOTHER METHOD.
   - B STATE THE NUMBER OF ELEMENTS THAT BELONG TO EACH SET IN QUESTION 2.
   - C IN HOW MANY WAYS CAN YOU DESCRIBE THE SETS IN QUESTION 2?
4. WHICH OF THE SETS IN QUESTION 2 HAVE
   - A NO ELEMENT
   - B A FINITE NUMBER OF ELEMENTS
   - C INFINITELY MANY ELEMENTS

3.1.1 Sets and Elements

Set: A set is a *well-defined* collection of objects.

When we say that a set is well-defined, we mean that, given an object, we are able to determine whether the object is in the set or not. For instance, "*the collection of all intelligent people in Africa*" cannot form a well-defined set, since we may not agree on who is an "*intelligent person*" and who is not.

The individual objects in a set are called the *elements* or the *members*. Repeating elements in a set does not add new elements to the set.

For example, the set $\{a, a\}$ is the same as $\{a\}$.

Notation: Generally, we use capital letters to name sets and represent their elements by small letters. The symbol $\in$ stands for the phrase 'is an element of' (or 'belongs to'). So, $x \in A$ is read as 'x is an element of A' or 'x belongs to A'. We write the statement 'x does not belong to A' as $x \notin A$.

Since the phrase '*the set of*' occurs so often, we use the symbol *brace* (or *curly bracket*) $\{\ \}$.

For instance, '*the set of all vowels in the English alphabet*' is written as

$\{$all vowels in the English alphabet$\}$ or $\{a, e, i, o, u\}$.

3.1.2 Description of Sets

A set may be described by three methods:

I Verbal method

We may describe a set in words. For instance,

a The set of all whole numbers less than ten. This can be written as $\{$all whole numbers less than ten$\}$.

b The set of all natural numbers. This can also be written as $\{$all natural numbers$\}$.

II The listing method (also called *roster* or *enumeration method*)

If the elements of a set can be listed, then we can describe the set by listing. The elements can be listed completely or partially as illustrated in the following example:

Example 1 Describe (express) each of the following sets using the listing method.

a The set whose elements are a, 2 and 7.

b The set of natural numbers less than 51.

c The set of whole numbers.

d The set of non-positive integers.

e The set of integers.

SOLUTION:

A FIRST LET US NAME THE SET BY A. THEN WE CAN DESCRIBE THE SET AS

$A = \{a, 2, 7\}$

B THE NATURAL NUMBERS LESS THAN 51 ARE 1, 2, 3, . . ., 50. SO, NAMING THE SET WE CAN EXPRESS B BY THE LISTING METHOD AS

$B = \{1, 2, 3, . . ., 50\}$

THE THREE DOTS AFTER THE ELEMENT 3 (CALLED AN ELLIPSIS) INDICATE ELEMENTS IN THE SET CONTINUE IN THAT MANNER UP TO AND INCLUDIN ELEMENT 50.

C NAMING THE SET OF WHOLE NUMBERS BY $\mathbb{W}$, WE CAN DESCRIBE IT AS

$\mathbb{W} = \{0, 1, 2, 3, . . . \}$

THE THREE DOTS INDICATE THAT THE ELEMENTS CONTINUE IN THE GIVEN P THERE IS NO LAST OR FINAL ELEMENT.

D IF WE NAME THE SET BY L, THEN WE DESCRIBE THE SET AS

$L = \{. . ., -3, -2, -1, 0\}$

THE THREE DOTS THAT PRECEDE THE NUMBERS INDICATE THAT ELEMENT FROM THE RIGHT TO THE LEFT IN THAT PATTERN AND THERE IS NO BEGINNIN

E YOU KNOW THAT THE SET OF INTEGERS IS DENOTED BY $\mathbb{Z}$ AND DESCRIBED BY

$\mathbb{Z} = \{. . ., -3, -2, -1, 0, 1, 2, 3, . . .\}$

WE USE THE **partial listing method**, IF LISTING ALL ELEMENTS OF A SET IS DIFFICULT O IMPOSSIBLE BUT THE ELEMENTS CAN BE INDICATED UNAMBIGUOUSLY BY LISTING A FEW

Exercise 3.1

1 DESCRIBE EACH OF THE FOLLOWING SETS USING A VERBAL METHOD:

A $A = \{5, 6, 7, 8, 9\}$ **B** $M = \{2, 3, 5, 7, 11, 13\}$

C $G = \{8, 9, 10, . . .\}$ **D** $E = \{1, 3, 5, . . ., 99\}$

2 DESCRIBE EACH OF THE FOLLOWING SETS USING THE LISTING METHOD (IF POSSIBL

A THE SET OF PRIME FACTORS OF 72.

B THE SET OF NATURAL NUMBERS THAT ARE LESS THAN 113 AND DIVISIBLE BY 5

C THE SET OF NON-NEGATIVE INTEGERS.

D THE SET OF RATIONAL NUMBERS BETWEEN $\sqrt{2}$ AND $\sqrt{5}$.

E THE SET OF EVEN NATURAL NUMBERS.

F THE SET OF INTEGERS DIVISIBLE BY 3.

G THE SET OF REAL NUMBERS BETWEEN 1 AND 3.

III The set-builder method (*also known as method of defining property*)

ACTIVITY 3.2

TO EACH DESCRIPTION GIVEN IN COLUMN A, MATCH A SET THAT SA FROM COLUMN B.

	A		B
1	$2 < x < 10$ AND $x \in \mathbb{N}$	**A**	$\{1, 2, 3, 4, 6, 12\}$
2	$x = 2n$ AND $n \in \mathbb{N}$	**B**	$\{-2\}$
3	$2x + 4 = 0$ AND x IS AN INTEGER	**C**	$\{2, 4, 6, \ldots\}$
4	$x \in \mathbb{N}$ AND 12 IS A MULTIPLE OF x	**D**	$\{3, 4, 5, 6, 7, 8, 9\}$

THE ABOVE ACTIVITY LEADS YOU TO THE THIRD USEFUL METHOD FOR DESCRIBING SETS, K the set-builder method.

FOR EXAMPLE, $A = \{3, 4, 5, 6, 7, 8, 9\}$ CAN BE DESCRIBED AS

$$A = \{x \mid x \in \mathbb{N} \text{ AND } 2 < x < 10\} \text{ OR } A = \{x \mid x \in \mathbb{N} \text{ AND } 2 < x < 10\}$$

NOTE THAT "SUCH THAT" MAY BE WRITTEN "$x \mid$" OR "$x:$".

HENCE WE READ THE ABOVE AS "SET A IS THE SET OF ALL ELEMENTS x SUCH THAT x IS A NATURAL NUMBER BETWEEN 2 AND 10"

NOTE THAT IN THE ABOVE SET A THE PROPERTIES THAT CHARACTERIZE THE ELEMENTS ARE $x \in \mathbb{N}$ AND $2 < x < 10$.

EXAMPLE 2 EXPRESS EACH OF THE FOLLOWING SETS USING THE SET-BUILDER METHOD:

A $\mathbb{N} = \{1, 2, 3, \ldots\}$ **B** A = {REAL NUMBERS BETWEEN

C B = {INTEGERS DIVISIBL **D** THE REAL SOLUTION SET OF $|x - 1| = 2$

SOLUTION:

A $\mathbb{N} = \{x \mid x \in \mathbb{N}\}$

B $A = \{x \mid x \in \mathbb{R} \text{ AND } 0 < x < 1\}$

NOTE THAT THIS SET CAN BE EXPRESSED AS $A = \{x \in \mathbb{R} \mid 0 < x < 1\}$

C $B = \{x \mid x = 3n, \text{ FOR SOME INTEGER } n\}$ OR $B = \{3n \mid n \in \mathbb{Z}\}$

D NAMING THE SET S, WE WRITE $S = \{x \mid x \in \mathbb{R} \text{ AND } |x - 1| = 2\}$

Exercise 3.2

1 WHICH OF THE FOLLOWING (ARE WELL DEFINED? YOUR ANS.

A $\{x \mid x$ IS AN INTERESTIN(B $\{x \mid x$ IS A GOOD STU.

C THE SET OF NATURAL NUMBERS L D $\{y \mid y$ IS A FACTOR O

2 WHICH OF THE FOLIARE TRUE AND WHICH ARE FALSE?

A $2 \in \{-1, 0, 1\}$ B $a \notin \{\{a, c\}\}$ C $6 \in$ {FACTORS O

3 DESCRIBE EACH OF THE FOLLOV

I THE LISTING MI II THE SET–BUILDER METH(

A THE SET OF LETTERS IN THE WORD

B THE SET REGIONAL STATES IN ETHIOPIA.

C THE SET OF WHOLE NUMBERS BETWE

D THE SET OF EVEN NUMBERS LE

E THE SET OF STUDENTS IN

F THE SET OF ALL ODD NATUR.

4 DESCRIBE EACH OF THE FOLL BY

I A VERBAL ME. II THE SET–BUILDER METH(

A $\{1, 2, 3,\ldots, 10\}$ B $\{1, 3, 5, 7,\ldots\}$ C $\{5, 10, 15, 20,\ldots\}$

D {TUESDAY, THURS E $\{2, 3, 5, 7, 11,\ldots\}$

3.2 THE NOTION OF SETS

3.2.1 Empty Set, Finite Set, Infinite Set, Subset, Proper Subset

ACTIVITY 3.3

1 HOW MANY ELEMENTEACH OF THE FOLLOWING SETS H

A $A = \{x \mid x$ IS A REAL NUMBER WHOSE SQUARE IS

B $C = \{x \mid x \in \mathbb{N}$ AND $2 < x < 11\}$ C $D = \{x \mid x \in \{1, 2, 3\}\}$

D $E = \{x \mid x$ IS AN INTE(E $X = \{2, 4, 6, \ldots\}$

2 COMPARE THE SETS C, D, E AND IN QUESTION 1 ABOVE WHICH S IS CONTAINED IN A?

OBSERVE FROM THE ABOVE ACTIVITY THAT A SET MAY HAVE NO ELEMENTS, A LIMIT ELEMENTS OR AN UNLIMITED NUMBER

A Empty set

Definition 3.1

A set that contains no elements is called an empty set, or null set.

An empty set is denoted by either ∅ or { }.

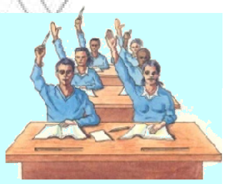
EXAMPLE 1

A IF A = $\{x \mid x$ IS A REAL NUMBER $x^2 = -1\}$, A = ∅ (WHY?)

B IF B = $\{x \mid x \neq x\}$, B = ∅. (WHY?)

B Finite and infinite sets

ACTIVITY 3.4



WHICH OF THE FOLLOWING HAVE A FINITE AND WHICH HAVE NUMBER OF ELEMENTS

1 A = $\{x \mid x \in \mathbb{R}$ AND $0 < x < 3\}$

2 C = $\{x \in \mathbb{N} \mid 7 < x < 7^{100}\}$

3 D = $\{x \in \mathbb{N} \mid x$ IS A MULTIPLE$\}$

4 E = $\{x \in \mathbb{Z} \mid 2 < x < 3\}$

5 M = $\{x \in \mathbb{N} \mid x$ IS DIVISIBLE BY $x < 101^4\}$

YOUR OBSERVATIONS FROM ABOVE ACTIVITY LEAD TO THE FOLLOWING DEFINITION

Definition 3.2

I A SET S IS CALLED finite, IF IT CONTAINS n ELEMENTS WHERE n IS SOME NON-NEGATIVE INTEGER.

II A SET S IS CALLED infinite, IF IT IS NOT FINITE.

Notation: IF A SET S IS FINITE, THEN WE DENOTE THE NUMBER OF ELEMENTS n (S).

EXAMPLE 2 IF S = {–1, 0, 1}, THEN n(S) = 3

USING THIS NOTATION, WE CAN SAY THAT A SET S IS FINITE IF n (S) = 0 OR n (S) IS A NATURAL NUMBER.

EXAMPLE 3 FIND n(S) IF:

A S = $\{x \in \mathbb{R} \mid x^2 = -1\}$ **B** S = $\{x \in \mathbb{N} \mid x$ IS A FACTOR OF 108$\}$

SOLUTION:

A n (S) = 0 **B** n (S) = 12

EXAMPLE 4

A LET E = {2, 4, 6…}. E IS INFINITE.

B LET T = {x | x IS A REAL NUMBER AND x < 1}. T IS INFINITE.

C Subsets

ACTIVITY 3.5

WHAT IS THE RELATION BETWEEN EACH OF THE FOLLOWING PAIRS OF SETS?

1 M = {ALL STUDENTS IN YOUR CLASS WHOSE NAMES BEGIN WITH A VOWEL};

N = {ALL STUDENTS IN YOUR CLASS WHOSE NAMES BEGIN WITH

2 A = {1, 3, 5, 7}; B = {1, 2, 3, 4, 5, 6, 7, 8}

3 E = {$x \in \mathbb{R} \mid (x - 2)(x - 3) = 0$}; F = {$x \in \mathbb{N} \mid 1 < x < 4$}

Definition 3.3

Set A is a subset of set B, denoted by A $\subseteq$ B, if each element of A is an element of B.

Note: IF A IS NOT A SUBSET OF B, THEN WE DENOTE THIS BY A $\not\subseteq$ B.

EXAMPLE 5 LET $\mathbb{Z}$ = {$x \mid x$ IS AN INTEGER} AND $\mathbb{Q}$ = {$x \mid x$ IS A RATIONAL NU

SINCE EACH ELEMENT OF $\mathbb{Z}$ IS ALSO AN ELEMENT OF $\mathbb{Q}$, THEN $\mathbb{Z} \subseteq \mathbb{Q}$

EXAMPLE 6 LET G = {–1, 0, 1, 2, 3} AND H = {0, 1, 2, 3, 4, 5

–1 $\in$ G BUT –1 $\notin$ H, HENCE G $\not\subseteq$ H.

Note: FOR ANY SET A

I $\varnothing \subseteq$ A **II** A $\subseteq$ A

Group Work 3.1

GIVEN A = {a, b, c}

1 LIST ALL THE SUBSE

2 HOW MANY SUBSETS HAVE YO

FROM GROUP WORK 3.1, YOU CAN MAKE THE FOLLOWING DEFINITION.

Definition 3.4

Let A be any set. The power set of A, denoted by P(A), is the set of all subsets of A. That is, P(A) = {S | S ⊆ A}

EXAMPLE 7 LET M = {–1, 1}. THEN SUBSETS OF M ARE ∅, {–1}, {1} AND M

THEREFORE P(M) = {∅, {–1}, {1}, M}

D Proper subset

LET A = {–1, 0,1} AND B = {–2, –1, 0, 1}. FROM THESE SETS, WE SEE THAT A ⊆ B BUT B ⊈ A. THIS SUGGESTS THE DEFINITION OF A PROPER SUBSET STATED BELOW.

Definition 3.5

Set A is said to be a proper subset of a set B, denoted by A ⊂ B, if A is a subset of B and B is not a subset of A.

THAT IS, A ⊂ B MEANS A ⊆ B BUT B ⊈ A.

Note: FOR ANY SET A, A IS NOT A PROPER SUBSET OF ITSELF.

ACTIVITY 3.6

GIVEN A = {–1, 0, 1}.

I LIST ALL PROPER SUBSETS

II HOW MANY PROPER SUBSETS OF A HAVE YOU FOUND?

YOU WILL NOW INVESTIGATE THE RELATIONSHIP BETWEEN THE NUMBER OF ELEMENTS OF A SET AND THE NUMBER OF ITS SUBSETS AND PROPER SUBSETS.

ACTIVITY 3.7

1 FIND THE NUMBER OF SUBSETS AND PROPER SUBSETS OF EACH OF THE FOLLOWING SETS

A A = ∅ **B** B = {0} **C** C = {–1, 0} **D** D = {–1, 0, 1}

2 COPY AND COMPLETE THE FOLLOWING TABLE:

	Set	No. of elements	Subsets	No. of subsets	Proper subsets	No. of proper subsets
A	$\varnothing$	0	$\varnothing$	$1 = 2^0$	-	$0 = 2^0 - 1$
B	$\{0\}$	1	$\varnothing, \{0\}$	$2 = 2^1$	$\varnothing$	$1 = 2^1 - 1$
C	$\{-1, 0\}$					
D	$\{-1, 0, 1\}$		$\varnothing, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{0, 1\}, \{-1, 1\}, \{-1, 0, 1\}$			$7 = 2^3 - 1$

YOU GENERALIZE THE RESULT OF THE ABOVE ACTIVITY IN THE FORM OF THE FOLLOWING FACT.

Fact: IF A SET A IS FINITE WITH n ELEMENTS, THEN

I THE NUMBER OF SUBSETS OF A IS 2^n AND

II THE NUMBER OF PROPER SUBSETS OF A IS $2^n - 1$.

Exercise 3.3

1 FOR EACH SET IN THE LEFT COLUMN, CHOOSE THE SETS FROM THE RIGHT COLUMN SUBSETS OF IT:

I $\{a, b, c, d\}$	**A** $\{\ \}$
II $\{o, p, k\}$	**B** $\{1, 4, 8, 9\}$
III SET OF LETTERS IN THE WOI	**C** $\{o, k\}$
IV $\{2, 4, 6, 8, 10, 12\}$	**D** $\{12\}$
	E $\{6\}$

2 **A** IF B = $\{0, 1, 2\}$, FIND ALL SUBSETS OF B.

B IF B = $\{0, \{1, 2\}\}$, FIND ALL SUBSETS OF B.

3 STATE WHETHER EACH OF THE FOLLOWING STATEMENTS IS TRUE OR FALSE. IF IT IS FALSE, JUSTIFY YOUR ANSWER.

A $\{1, 4, 3\} \subseteq \{3, 4, 1\}$ **B** $\{1, 3, 1, 2, 3, 2\} \not\subseteq \{1, 2, 3\}$

C $\{4\} \subseteq \{\{4\}\}$ **D** $\varnothing \subseteq \{\{4\}\}$

3.2.2 Venn Diagrams, Universal Sets, Equal and Equivalent Sets

A Venn diagrams

ACTIVITY 3.8

1 WHAT IS THE RELATION BETWEEN THE FOLLOWING PAIRS OF SETS?

A $\mathbb{W}$ = { 0, 1, 2,…} AND $\mathbb{N}$ = { 1, 2, 3,…}.

B $\mathbb{W}$ = {0, 1, 2,…} AND $\mathbb{Z}$ = {…,–3, –2, –1, 0, 1, 2,…}.

C $\mathbb{N}$ = { 1, 2, 3,…} AND $\mathbb{Z}$ = {…,–3, –2, –1, 0, 1, 2,…}.

D $\mathbb{Z}$ = {…,–3, –2, –1, 0, 1, 2,…} AND $\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z},\ b \neq 0 \right\}$.

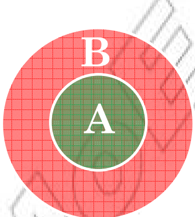
2 EXPRESS THE RELATION BETWEEN EACH PAIR USING A DIAGRAM.

3 EXPRESS THE RELATION OF ALL THE SETS $\mathbb{W}$, $\mathbb{N}$, $\mathbb{Z}$ AND $\mathbb{Q}$ USING ONE DIAGRAM.

COMPARE YOUR DIAGRAM WITH THE ACTIVITY 1.1 OF UNIT 1.

TO ILLUSTRATE VARIOUS RELATIONS THAT ARISE BETWEEN SETS, IT IS OFTEN HELPFUL TO USE A PICTORIAL REPRESENTATION CALLED A DIAGRAM NAMED AFTER JOHN VENN (1834 – 1883). THESE DIAGRAMS CONSIST OF RECTANGLES, CLOSED CURVES, USUALLY CIRCLES. THE ELEMENTS OF THE SETS ARE WRITTEN IN THEIR RESPECTIVE CIRCLES.

FOR EXAMPLE, THE RELATION '$A \subset B$' CAN BE ILLUSTRATED BY THE FOLLOWING DIAGRAM.



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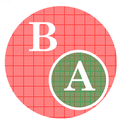


Figure 3.1

EXAMPLE 1 REPRESENT THE FOLLOWING PAIRS OF SETS USING VENN DIAGRAMS:

A A = {a, b, c, d}; B = {a, d}

B C = {2, 4, 6, 8,…}; D = {1, 3, 5, 7,…}

C E = $\{2^n \mid n \in \mathbb{N}\}$; F = $\{2n \mid n \in \mathbb{N}\}$

D A = {1, 3, 5, 7, 9}; B = { 2, 3, 5, 8}; C = { 1, 5, 7}

SOLUTION:

A

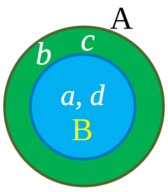


Figure 3.2

B

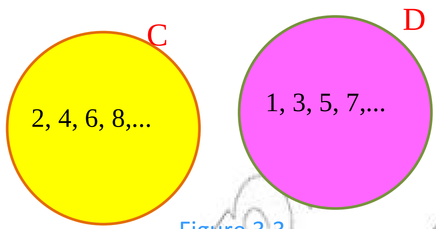


Figure 3.3

C

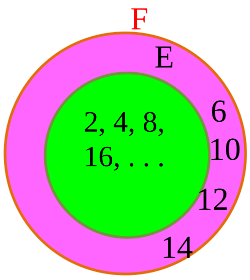


Figure 3.4

D

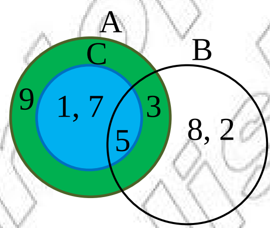


Figure 3.5

B Universal set

SUPPOSE AT A SCHOOL ASSEMBLY, THE FOLLOWING STUDENTS ARE ASKED TO STAY BE

G = {ALL GRADE 9 STUDENTS}.

I = {ALL STUDENTS INTERESTED IN A SCHOOL PLAY}.

R = {ALL CLASS REPRESENTATIVES OF EACH CLASS}.

EACH SET, G, I AND R IS A SUBSET OF S = {ALL STUDENTS IN THE SCHOOL}

IN THIS PARTICULAR EXAMPLE, S IS CALLED THE universal set.

SIMILARLY, A DISCUSSION IS LIMITED TO A FIXED SET OF OBJECTS AND IF ALL THE EL DISCUSSED ARE CONTAINED IN THIS SET, THEN THIS "OVERALL" SET IS CALLED THE universal set. WE USUALLY DENOTE THE UNIVERSAL SET BY U. DIFFERENT PEOPLE MAY CHOOS UNIVERSAL SETS FOR THE SAME PROBLEM.

EXAMPLE 2 LET R = {ALL RED COLOURED CARS IN EAST AFRICA}, T = {ALL TOYOTA CARS IN EAST AFRICA}

I CHOOSE A UNIVERSAL SET FOR R AND T.

II DRAW A VENN DIAGRAM TO REPRESENT THE SETS U, R AND T.

SOLUTION:

I THERE ARE DIFFERENT POSSIBILITIES FOR U. TWO OF THESE ARE:

U = {ALL CARS} OR U = {ALL WHEELED VEHICLES}

ii IN BOTH CASES, THE VENN DIAGRAM OF SETS U, R AND T IS

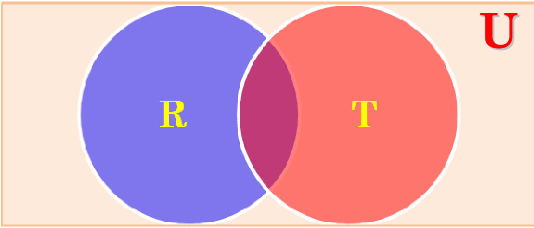


Figure 3.6

Exercise 3.4

1 DRAW VENN DIAGRAMS TO ILLUSTRATE THE RELATIONSHIPS BETWEEN THE FOLLOWIN SETS:

a A = {1, 9, 2, 7, 4}; L = {4, 9, 8, 2}

b B = {THE VOWELS IN ENGLISH ALPHABET}

M = {THE FIRST LETTERS OF THE ENGLISH ALPHAE

c C = {1, 2, 3, 4, 5}; M = {6, 9, 10, 8, 7}

d F = {3, 7, 11, 5, 9}; O = {ALL ODD NUMBERS BETWEEN

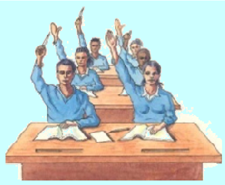
2 FOR EACH OF THE FO, DRAW A VENN DIAGRAM TO ILLUSTRATE THE RELATIONSHIP BETWEEN THE SE

a U = {ALL ANIMAL; C = {ALL COWS}; G ={ALL GO

b U = {ALL PEOF; M = {ALL MALES}; B = {ALL BO

C Equal and equivalent sets

ACTIVITY 3.9



FROM THE FOLLOWING PAIR IDENTIFY THOSE:

1 THAT ARE THE SAME NUMBER OF

2 THAT HAVE EXACTLY THE SAI

a A = {1, 2}; B = $\{x \in \mathbb{N} \mid x < 3\}$

b E = {–1, 3}; F = $\left\{\frac{1}{2}, \frac{1}{3}\right\}$

c R = {1, 2, 3}; S = $\{a, b, c\}$

d G = $\{x \in \mathbb{N} \mid x$ IS A FACTOR ; H = $\{x \in \mathbb{N} \mid 6$ IS A MULTIPL $x\}$

e X = {1, 1, 3, 2, 3, 1}; Y = {1, 2, 3}

I Equality of sets

LET US INVESTIGATE THE RELATIONSHIP BETWEEN THE FOLLOWING TWO SETS;

$$E = \{x \in \mathbb{R} \mid (x-2)(x-3) = 0\} \text{ AND } F = \{x \in \mathbb{N} \mid 1 < x < 4\}.$$

BY LISTING COMPLETELY THE ELEMENTS OF EACH SET, WE HAVE E = {2, 3} AND F = {2, 3

WE SEE THAT E AND F HAVE EXACTLY THE SAME ELEMENTS. SO THEY ARE EQUAL.

IS E $\subseteq$ F? Is F $\subseteq$ E?

Definition 3.6

Given two sets A and B, if every element of A is also an element of B and if every element of B is also an element of A, then the sets A and B are said to be equal. We write this as A = B.

$\therefore$ A = B, if and only if A $\subseteq$ B and B $\subseteq$ A.

EXAMPLE 3 LET A = {1, 2, 3, 4} AND B = {1, 4, 2, 3}.

A = B, SINCE THESE SETS CONTAIN EXACTLY THE SAME ELEMENTS.

Note: IF A AND B ARE NOT EQUAL, WE WRITE A $\neq$ B.

EXAMPLE 4 LET C = {–1, 3, 1} AND D = {–1, 0, 1, 2}.

C $\neq$ D, BECAUSE 2 $\in$ D, BUT 2 $\notin$ C.

II Equivalence of sets

CONSIDER THE SETS A = {a, b, c} AND B = {2, 3, 4}. EVEN THOUGH THESE TWO SETS ARE NOT EQUAL, THEY HAVE THE SAME NUMBER OF ELEMENTS. SO, FOR EACH MEMBER OF SE FIND A PARTNER IN SET A.

A = {a, b, c}

B = {2, 3, 4}

THE DOUBLE ARROW SHOWS HOW EACH ELEMENT OF A SET IS MATCHED WITH AN ANOTHER SET. THIS MATCHING COULD BE DONE IN DIFFERENT WAYS, FOR EXAMPLE:

A = {a, b, c}

B = {2, 3, 4}

NO MATTER WHICH WAY WE MATCH THE SETS, EACH ELEMENT OF A IS MATCHED WITH ELEMENT OF B AND EACH ELEMENT OF B IS MATCHED WITH EXACTLY ONE ELEMENT C THAT THERE IS A ONE–TO–ONE CORRESPONDENCE BETWEEN A AND B.

Definition 3.7

Two sets A and B are said to be equivalent, written as A↔B (or A~B), if there is a one-to-one correspondence between them.

Observe that two finite sets A and B are equivalent, if and only if

n (A) = n (B)

EXAMPLE 5 LET A = {$\sqrt{2}$, e, } AND B = {1, 2, 3}.

SINCE n(A) = n(B), A AND B ARE EQUIVALENT SETS AND WE WRITE

A ↔ B.

NOTE THAT EQUAL SETS ARE ALWAYS EQUIVALENT SINCE EACH ELEMENT CAN BE ITSELF, BUT EQUIVALENT SETS ARE NOT NECESSARILY EQUAL. FOR EXAMPLE,

{1, 2} ↔ {a, b} BUT {1, 2} ≠ {a, b}.

Exercise 3.5

WHICH OF THE FOLLOWING PAIRS REPRESENT EQUAL SETS AND WHICH OF THEM REPRE EQUIVALENT SETS?

1 {a, b} AND {2, 4}

2 {∅} AND ∅

3 $\{x \in \mathbb{N} \mid x < 5\}$ AND {2, 3, 4, 5}

4 {1, {2, 4}} AND {1, 2, 4}

5 $\{x \mid x < x\}$ AND $\{x \in \mathbb{N} \mid x < 1\}$

3.3 OPERATIONS ON SETS

THERE ARE OPERATIONS ON SETS AS THERE ARE OPERATIONS ON NUMBERS. LIKE THE ADDTION AND MULTIPLICATION ON NUMBERS, INTERSECTION AND UNION ARE OPERATIO

3.3.1 Union, Intersection and Difference of Sets

A Union of sets

Definition 3.8

The union of two sets A and B, denoted by A∪B and read "A union B" is the set of all elements that are members of set A or set B or both of the sets. That is, A∪B = $\{x \mid x \in A \text{ or } x \in B\}$

THE *red* SHADED REGION OF THE DIAGRAM ON THE RIGHT REPRES$\cup$ B.

AN ELEMENT COMMON TO BOTH SETS IS LISTE THE UNION. FOR EXAMPLE, IF A = $\{a, b, c, d, e\}$ AND B = $\{c, d, e, f, g\}$, THEN

A$\cup$B = $\{a, b, c, d, e, f, g\}$.

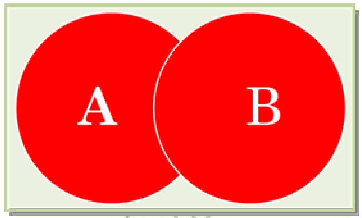


Figure 3.7

EXAMPLE 1

A $\{a, b\} \cup \{c, d, e\} = \{a, b, c, d, e\}$

B $\{1, 2, 3, 4, 5\} \cup \varnothing = \{1, 2, 3, 4, 5\}$

Properties of the union of sets

ACTIVITY 3.10

LET A = {1, 2, 3, 4}, B = {2, 4, 6, 8} AND C = {3, 4, 5, 6}.

1 FIND **A** A$\cup$B **B** B$\cup$A

WHAT IS THE RELSHI BETWEEN A$\cup$B AND B$\cup$A?

2 FIND **A** A$\cup$B **B** (A$\cup$B)$\cup$C **C** B$\cup$C **D** A$\cup$(B$\cup$C)

WHAT IS THE RELSHI BETWEEN (A$\cup$B)$\cup$C AND A$\cup$(B$\cup$C)?

3 FIND A$\cup\varnothing$, WHAT IS THE RELSHIP BETWEEN A$\cup\varnothing$ AND A?

THE ABOVE ACTIVITY LEADS YOU TO THE FOLLOWING PROPERTIES:

FOR ANY SETS A, B AND C

1 COMMUTATIVE PRO A$\cup$B = B$\cup$A

2 ASSOCIATIVE PRO (A$\cup$B)$\cup$C = A$\cup$(B$\cup$C)

3 IDENTITY PROPER A$\cup\varnothing$ = A

Exercise 3.6

1 GIVEN A = {1, 2, {3}} B = {2, 3} AND C = {{3}, 4}, FIND:

A A$\cup$B **B** B$\cup$C **C** A$\cup$C

D A$\cup$(B$\cup$C) **E** (A$\cup$B)$\cup$C

2 STATE WHETHER EACH OF THE FOLLOWING STATEMEN:

A IF $x \in$ A AND $x \notin$ B, THEN $x \notin$ (A$\cup$B). **B** IF $x \in$ (A$\cup$B) AND $x \notin$ A, THEN $x \in$ B.

C IF $x \notin$ A AND $x \notin$ B, THEN $x \notin$ (A$\cup$B). **D** FOR ANY SET A, A$\cup$A = A.

E FOR ANY SET A, A$\cup\varnothing$ = A. **F** IF A $\subseteq$ B, THEN A$\cup$B = B.

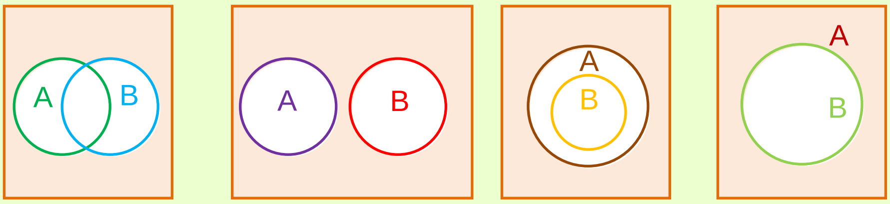
G FOR ANY TWO SETS A AND B, A ⊆ (A∪B) AND B ⊆ (A∪B).

H FOR ANY THREE SETS A, B, C, IF A ⊆ B, AND B ⊆ C, THEN A ∪ B = C.

I FOR ANY THREE SETS A, IF A∪B = C, THEN B ⊆ C.

J IF A∪B = ∅, THEN A = ∅ AND B = ∅.

3 USING COPIES OF THE VENN DIAGRAMS BELOW, SHADE A∪B.



A B C D

Figure 3.8

B Intersection of sets

ACTIVITY 3.11

CONSIDER THE SETS G = {2, 4, 6, 8, 10, 12} AND H = {1, 2, 3, 4, 5}.

A DRAW A VENN DIAGRAM THAT SHOWS THE RELATIONSHIP BETWEEN THE TWO SETS.

B SHADE THE REGION COMMON TO G AND H AND FIND THEIR COMMON ELEMENTS.

Definition 3.9

The intersection of two sets A and B, denoted by A∩B and read as "*A intersection B*", is the set of all elements common to both set A and set B. That is, $A \cap B = \{ x \mid x \in A \text{ and } x \in B\}$.

USING THE VENN DIAGRAM, A ∩ B IS REPRESENTED BY THE *blue* SHADED REGION:

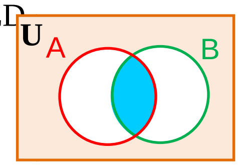


Figure 3.9

EXAMPLE 2 LET S = {*a*, *b*, *c*, *d*} AND T = {*b*, *d*, *g*}. THEN S ∩ T = {*b*, *d*}.

EXAMPLE 3 LET V = {2, 4, 6,...} (MULTIPLES OF 2) AND

W = {3, 6, 9,...} (MULTIPLES OF 3).

THEN V ∩ W = {6, 12, 18,...}, THAT IS, MULTIPLES OF 6.

EXAMPLE 4 LET A = {1, 2, 3} AND B = {5 7, 8}, THEN A ∩ B = ∅.

Definition 3.10

Two or more sets are disjoint if they have no common element.

A and B are disjoint, if and only if A ∩ B = ∅.

IN THE VENN DIAGRAM, THE SETS A AND I

HERE A ∩ B = ∅

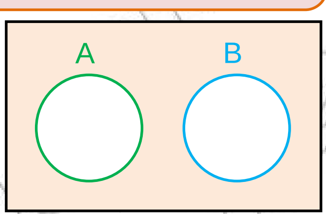


Figure 3.10

Properties of the intersection of sets

ACTIVITY 3.12

LET U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} BE THE UNIVERSAL SET AND I

A = {0, 2, 3, 5, 7}, B = {0, 2, 4, 6, 8} ANI

C = {x | x IS A FACTOR (

1 FIND **A** A∩C **B** C∩A

WHAT IS THE RELATIONSHIP BETWEEN A∩C AND C∩A?

2 FIND **A** A∩B **B** (A∩B) ∩C **C** B∩C **D** A∩(B∩C)

WHAT IS THE RELATIONSHIP BETWEEN (A∩B) ∩C AND A∩(B∩C)?

3 FIND A∩U. WHAT IS THE RELATIONSHIP BETWEEN A∩U AND A?

THE ABOVE ACTIVITY LEADS YOU TO THE FOLLOWING PROPERTIES:

FOR ANY SETS A, B AND C AND THE UNIVERSAL SET U

1 COMMUTATIVE PRC A∩B = B∩A.

2 ASSOCIATIVE PROI (A∩B)∩C = A∩(B∩C).

3 IDENTITY PROPEF A∩U = A.

Exercise 3.7

1 GIVEN A = {a, b, {c}}, B = {b, c} AND C = {{c}, d}, FIND:

A A∩B **B** A∩C **C** B∩C **D** A∩(B∩C)

2 STATE WHETHER EACH OF THE FOLLOWING STATEMENTS IS TRUE OR FAL

A IF $x \in$ A AND $x \notin$ B, THEN $x \in$ (A∩B). **B** IF $x \in$ (A∩B), THEN $x \in$ A AND $x \in$ B.

C IF $x \notin$ A AND $x \in$ B, THEN $x \in$ (A∩B). **D** FOR ANY SET A, A∩A = A.

E IF A ⊆ B, THEN A∩B = A.

F FOR ANY TWO SETS A AND B, $A \cap B \subseteq A$ AND $A \cap B \subseteq B$.

G IF $A \cap B = \varnothing$, THEN $A = \varnothing$ OR $B = \varnothing$.

H IF $(A \cup B) \subseteq A$, then $B \subseteq A$.

I IF $A \subseteq B$, THEN $A \cap B = B$.

J IF $A \subseteq B$, THEN $A \cap B = \varnothing$.

K IF $A \subseteq B$, THEN $B' \subseteq A'$.

3 IN EACH VENN DIAGRAM BELOW, SHADE $(A \cap B) \cap C$.

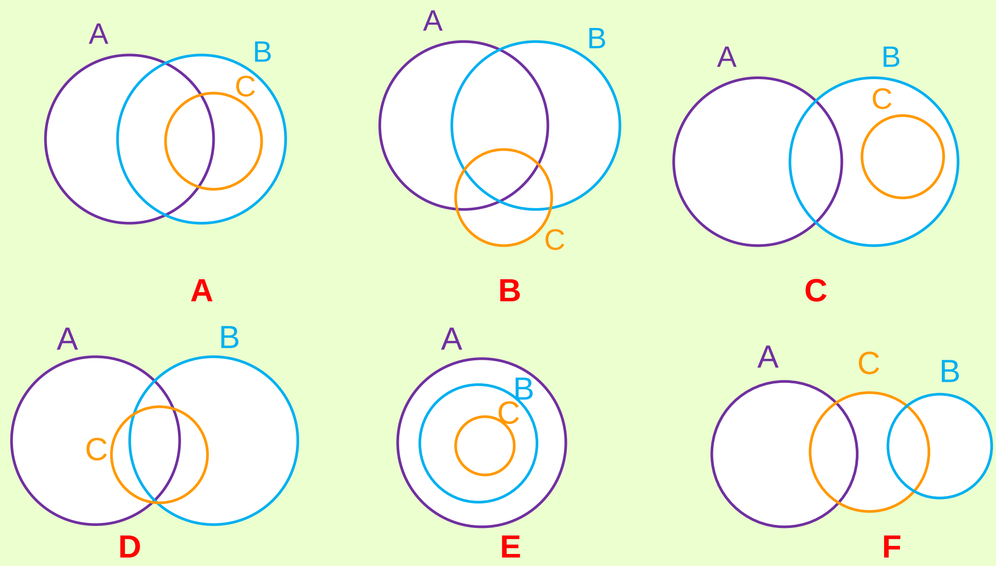


Figure 3.11

C Difference and symmetric difference of sets

I The relative complement (or difference) of two sets

GIVEN TWO SETS A AND B, THE COMPLEMENT OF B RELATIVE TO A (OR THE DIFFERENCE AND B) IS DEFINED AS FOLLOWS.

> **Definition 3.11**
>
> The relative complement of a set B with respect to a set A (or the difference between A and B), denoted by A – B, read as "A difference B", is the set of all elements in A that are not in B.
>
> That is, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

Note: A – B IS SOMETIMES DENOTED BY A\B (READ AS 'A LESS B').

A – B AND A\B ARE USED INTERCHANGEABLY.

USING A VENN DIAGRAM, A\B CAN BE REPRESENTED BY SHADING THE REGION IN A WHICH IS N

A\B IS SHADED IN *light green*

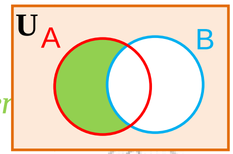


Figure 3.12

EXAMPLE 5 IF A = {x, y, z, w} AND B = { a, b, x, y}, THEN FIND:

A THE COMPLEMENT OF B REL **B** B\A **C** B\B

SOLUTION:

A NOTE THAT FINDING "THE COMPLEMENT OF B RELATIVE TO A" I "THE RELATIVE COMPLEMENT OF B WITH RESPEC A\B.

SO, A\B = {z, w}.

B B\A = { a, b}.

C B\B = ∅.

ACTIVITY 3.13

LET A = {0, 2, 3, 5, 7}, B = {0, 2, 4, 6, 8} AND C = {1, 2, 3, 6}. FIND:

A A\B **B** B\A **C** (A\B)\C **D** A\(B\C)

FROM THE RESULTS OF THE ABOVE ACTIVITY, WE CAN CONCLUDE THAT THE RELATIVE COMI SETS IS NEITHER COMMUTATIVE NO

II The complement of a set

LET U = {ALL HUMAN BEINGS} AND F = {A

THE VENN DIAGRAM OF THESE TWO SHOWN. TH YELLOW SHADED REGION U (BUT OUTSIDE F) IS CALL **complement** OF F, DENOTED F'.

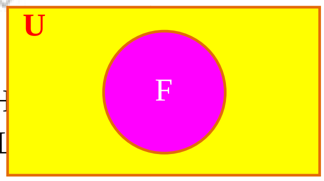


Figure 3.13

IT REPRESENTS ALL HUMAN BEINGS THAT ARE NOT FEMALE. THE MEMBERS OF F' ARE ALL THOSE MEMBERS OF U THAT ARE NOT ME

Definition 3.12

Let A be a subset of a universal set U. The complement (or absolute complement) of A, denoted by A', is defined to be the set of all elements of U that are not in A.

i.e., A' = {x | x ∈ U and x ∉ A}.

USING A VENN DIAGRAM, WE CAN REPRESENT A' BY THE SHA AS SHOWN IN FIGURE 3.14.

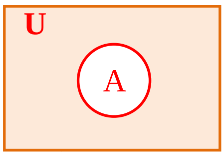


Figure 3.14

NOTE THAT FOR ANY SET A AND UN,

A′ = U\A

EXAMPLE 6 IN COPIES OF THE VENN DIAGRAM ON THE RIGHT, SHADE

A $A \setminus B$ **B** $(A \cap B)'$

C $A \cap B'$ **D** $A' \cup B'$

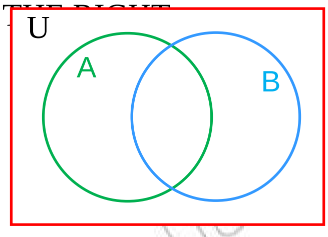


Figure 3.15

SOLUTION:

A SINCE $A \setminus B$ IS THE SET OF ALL ELEMENTS IN A THAT ARE NOT IN B, WE SHADE THE REGION IN A THAT IS NOT PART OF B (*shaded in green in* FIGURE 3.16).

A – B IS SHADED

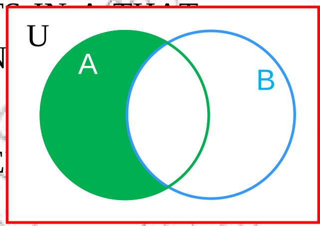
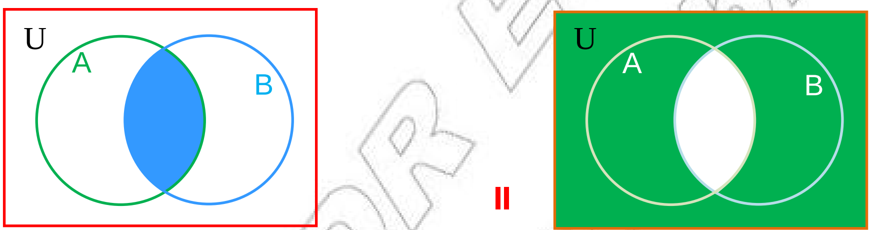


Figure 3.16

B FIRST WE SHADE THE REGION $A \cap B$; THEN $(A \cap B)'$ IS THE REGION OUTSIDE $A \cap B$.

I



II

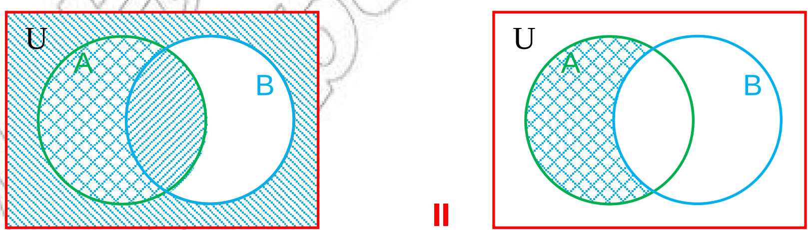
Figure 3.17

$A \cap B$ IS THE SHADED (*blue*) REGION. $(A \cap B)'$ IS THE *green* SHADED REGION.

C FIRST WE SHADE A WITH STROKES THAT SLANT UPWARD TO THE RIGHT (///) AND THEN SHADE B' WITH STROKES THAT SLANT DOWNWARD TO THE RIGHT (\\\\\\).

THEN $A \cap B'$ IS THE CROSS-HATCHED REGION.

I



II

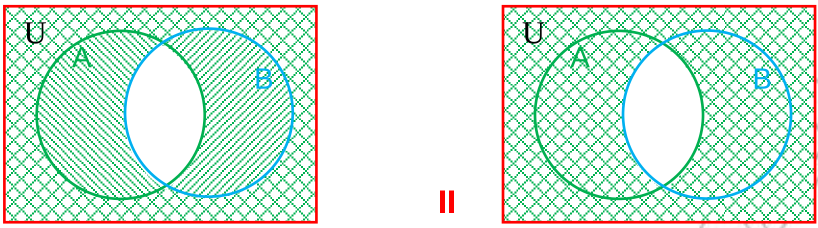
Figure 3.18

A AND B' ARE SHADED $A \cap B'$ IS SHADED

NOTE THAT THE REGION OF $A \setminus B$ IS THE SAME AS THE REGION OF $A \cap B'$.

D FIRST WE SHADE A', THE REGION OUTSIDE A, WITH STROKES THAT SLANT UPWARD TO THE RIGHT (///) AND THEN SHADE B' WITH STROKES THAT SLANT DOWNWARD TO THE RIGHT (\\\\\\)

THEN A'∪B' IS THE TOTAL SHADE

I 

II

Figure 3.19

A' OR B' ARE SHA A'∪B' IS SHAD

NOTE THAT THE REGION (A∩B)' IS THE SAME AS THE REGION A'∪B'.

Note: WHEN WE DRAW TWO OVERLAPPING CIRCLES IN A UNIVERSAL SET, FOUR REGIONS ARE FORMED. EVERY ELEMENT OF THE U IS IN EXACTLY ONE OF THE REGIONS.

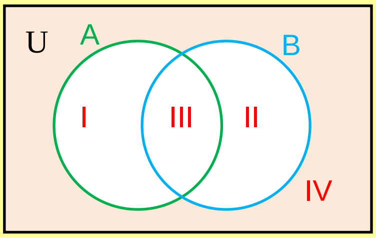


Figure 3.20

I IN A AND NOT IN B (A\B)

II IN B AND NOT IN A (B\A)

III IN BOTH A AND B (A∩B)

IV IN NEITHER A NOR B (A∪B)')

FROM ACTIVITY 3.13 AND THE ABOVE EXAMPLES, YOU GENERALIZE AS FOLLOWS:

FOR ANY TWO SETS A, B THE FOLLOWING PROPERTIES:

1 A\B = A ∩ B' **2** (A ∩ B)' = A' ∪ B' **3** (A\B)∪B = A∪B

ACTIVITY 3.14

1 IN COPIES OF THE SAME VENN DIAGRAM USED IN EXAMPLE 6, SHADE

A (A∪B)' **B** A'∩B'

2 GENERALIZE THE RESULT YOU QUESTION 1.

HISTORICAL NOTE:

Augustus De Morgan (1806-1871)

Augustus De Morgan was the first professor of mathematics at University College London and a cofounder of the London Mathematical Society.

De Morgan formulated his laws during his study of symbolic logic. De Morgan's laws have applications in the areas of set theory, mathematical logic and the design of electrical circuits.

Group Work 3.2

1 COPY FIGURE 3.21 AND SHADE THE REGION THAT REPR
OF THE FOLLOWING

A $(A\cup B)'$

B $A'\cup B'$

C $(A\cap B)'$

D $A'\cap B'$

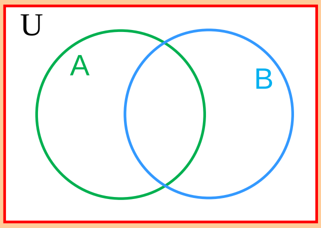


Figure 3.21

2 DISCUSS WHAT YOU HAVE OBSERVED FROM QUESTION 1

THE ABOVE GROUP WORK LEADS YOU TO THE FOLLOWING LAW CALLED De Morgan's law

Theorem 3.1 De Morgan's law

For any two sets A and B

1 $(A\cap B)' = A'\cup B'$ **2** $(A\cup B)' = A'\cap B'$

Exercise 3.8

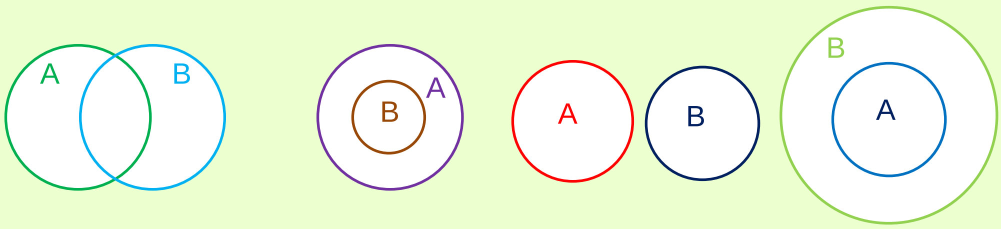
1 GIVEN $A = \{a, b, c\}$ AND $B = \{b, c, d, e\}$ FIND:

A THE RELATIVE COMPLEMENT OF A WITH RESPECT TO B.

B THE COMPLEMENT OF B RELATIVE TO A.

C THE COMPLEMENT OF A RELATIVE TO B.

2 IN EACH OF THE VENN DIAGRAMS GIVEN BELOW, SHADE A\ B.



A B C D

Figure 3.22

3 DETERMINE WHETHER EACH OF THE FOLLOWING STATEMENTS IS TRUE OR FALSE:

A IF $x\in A$ AND $x\notin B$ THEN $x\in (B\backslash A)$

B IF $x\in (A\backslash B)$ THEN $x\in A$

C $B\backslash A \subseteq B$, FOR ANY TWO SETS A AND B

D (A\B) ∩ (A∩B) ∩ (B\A) = ∅, FOR ANY TWO SETS A A

E IF A\B = ∅ THEN A ∅ AND B = ∅

F IF A ⊆ B THEN \B = ∅

G IF A∩B = ∅ THEN (\B) = A

H (A\B) ∪ B = A∪B, FOR ANY TWO SETS A AND B

I A∩A' = ∅

4 LET U = {1, 2, 3, . . ., 8, 9} B THE UNIVERSAL SET AND A = {, B = {2, 4, 6, 8} AND C = { 3, 4, 5, 6} LIST THE ELEME EACH OF THE FOLLO

A	A'	**B**	B'	**C**	(A∪C)'	**D**	(A\B)'		
E	A'∩B'	**F**	(A∪B)'	**G**	(A')'	**H**	B\C	**I**	B∩C'

III The symmetric difference between two sets

ACTIVITY 3.15

LET A = {*a*, *b*, *c*, *d*} AND B = {*b*, *d*, *e*}. THEN FIND:

A A∩B **B** A∪B **C** A\B

D B\A **E** (A∪B)\(A∩B) **F** (A\B) ∪ (B\A)

COMPARE THE RESULT AND F.

WHAT CAN YOU CONCLUDE YOUR ANSWERS?

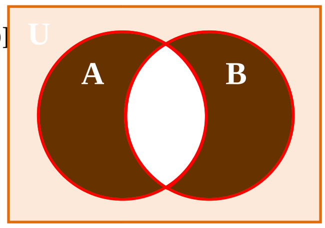
THE RESULT OF THE ACTIVITY ABOVE LEADS YOU TO STATE THE FOLLOWING

Definition 3.13

Let A and B be any two sets. The symmetric difference between A and B, denoted by A∆B, is the set of all elements in A∪B that are not in A∩B. That is A∆B = {*x* | *x* ∈ (A∪B) and *x* ∉ (A∩B)}

or A∆B = (A∪B)\(A∩B).

USING A VENN DIAGRAM, A∆B IS ILLUSTRATED BY SHADING THE REGION IN A∪B THAT IS NOT PART A∩B AS SHOWN.



A∆B IS THE SHADED *dark brown region.*

Figure 3.23

FROM ACTIVITY 3.15 AND THE ABOVE VENN DIAGRAM, YOU OBSERVE THAT

A∆B = (A∪B)\ (A∩B) = (A\B) ∪ (B\A).

Note: IF A ∩B = ∅ THEN A∆B = A ∪ B.

EXAMPLE 7 LET A = {–1, 0, 1} AND B = {1, 2}. FIND AΔB.

SOLUTION: $A\cup B = \{-1, 0, 1, 2\}$; $A\cap B = \{1\}$

$\therefore\ A\Delta B = (A\cup B)\backslash(A\cap B) = \{-1, 0, 2\}$

EXAMPLE 8 LET A = $\{a, b, c\}$ AND B = $\{d, e\}$. FIND AΔB.

SOLUTION: $A\cup B = \{a, b, c, d, e\}$; $A\cap B = \varnothing$

$\therefore\ A\Delta B = (A\cup B)\backslash\varnothing = A\cup B = \{a, b, c, d, e\}$

Distributivity

Group Work 3.3

1 GIVEN SETS A, B AND C, SHADE THE REGION THAT
EACH OF THE FOLLOWING

A $A\cup(B\cap C)$

B $(A\cup B)\cap(A\cup C)$

C $A\cap(B\cup C)$

D $(A\cap B)\cup(A\cap C)$

2 DISCUSS WHAT YOU HAVE OB
FROM QUESTION 1

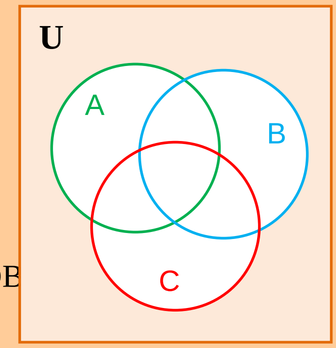


Figure 3.24

AS YOU MAY HAVE NOTICED FROM THE ABOVE FIGURE, THE FOLLOWING DISTRIBUTIVE PROPERTIES ARE TRUE:

Distributive properties

FOR ANY SETS A, B AND C

1 UNION IS DISTRIBUTIVE OVER THE INTERSECTION OF SETS.
I.E., $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$.

2 INTERSECTION IS DISTRIBUTIVE OVER THE UNION OF SETS.
I.E., $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$.

Exercise 3.9

1 IF $A\cap B = \{1, 0, -1\}$ AND $A\cap C = \{0, -1, 2, 3\}$, THEN FIND $A\cap(B\cup C)$.

2 SIMPLIFY EACH OF THE FOLLOWING BY USING VENN DIAGRAM OR ANY OTHER PRO

A $A\cap(A\cup B)$ B $P'\cap(P\cup Q)$

C $A\cap(A'\cup B)$ D $P\cup(P\cap Q)$

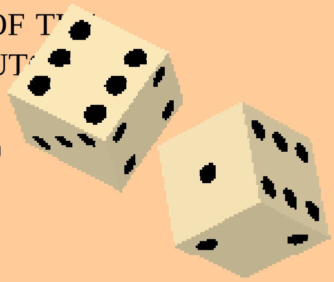
3.3.2 Cartesian Product of Sets

IN THIS SUBSECTION, YOU WILL LEARN HOW TO FORM SETS OF ORDERED PAIRS FROM TWO GIVEN SETS BY TAKING THE CARTESIAN PRODUCT OF THE SETS (NAMED AFTER THE MATHEMATICIAN Rene Descartes).

Group Work 3.4

A SIX-SIDED DIE (A CUBE) HAS FACES MARKED WITH NUMBERS 1,2,3,4,5 AND 6 RESPECTIVELY.

TWO SUCH DICE ARE THROWN AND THE NUMBERS ON THE UPPER FACES ARE RECORDED. FOR EXAMPLE, (6, 1) MEANS THAT THE NUMBER ON THE UPPER FACE OF THE FIRST DIE IS 6 AND THAT OF THE SECOND DIE IS 1. WE CALL THESE ORDERED PAIRS, THE OUTCOMES OF THE THROW OF OUR DICE.



LIST THE SET OF ALL POSSIBLE OUTCOMES OF THROWING SUCH THAT THE TWO NUMBERS

- I A: ARE BOTH EVEN.
- II B: ARE BOTH ODD.
- III C: ARE EQUAL.
- IV D: HAVE A SUM EQUAL TO 7.
- V E: HAVE A SUM EQUAL TO 1.
- VI F: HAVE AN EVEN SUM.
- VII G: HAVE THE FIRST NUMBER 1 AND THE SECOND EVEN.
- VIII H: HAVE A SUM LESS THAN 12.

FOR EXAMPLE, A= {(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}.

THE ACTIVITY OF THIS GROUP WORK LEADS YOU TO LEARN ABOUT THE SETS WHOSE ELEMENTS ARE ORDERED PAIRS.

Ordered pair

AN *ordered pair* IS AN ELEMENT (x, y) FORMED BY TAKING x FROM ONE SET AND y FROM ANOTHER SET. IN (x, y), WE SAY THAT x IS THE first ELEMENT AND y IS THE second ELEMENT.

SUCH A PAIR IS ORDERED IN THE SENSE THAT (x, y) AND (y, x) ARE NOT EQUAL UNLESS $x = y$.

Equality of ordered pairs

$(a, b) = (c, d)$, IF AND ONLY IF $a = c$ AND $b = d$.

EARLIER ALSO WE HAVE DISCUSSED ORDERED PAIRS WHEN WE REPRESENTED POINTS IN THE CARTESIAN COORDINATE SYSTEM. A POINT P IN THE PLANE CORRESPONDS TO AN ORDERED PAIR (a, b) WHERE a IS THE x-COORDINATE AND b IS THE y-COORDINATE OF THE POINT P.

EXAMPLE 1 A WEATHER BUREAU RECORDED HOURLY TEMPERATURES AS SHOWN IN THI FOLLOWING TABLE.

Time	9	10	11	12	1	2	3
Temp	61	62	65	69	68	72	76

THIS DATA ENABLES US TO MAKE SEVEN SENTENCES OF THE FORM:

AT x O'CLOCK THE TEMPERATURE WAS y DEGREES.

THAT IS, USING THE ORDERED PAIR (x, y), THE ORDERED PAIR (9, 61) MEANS.

At 9 *o'clock the temperature was* 61 *degrees.*

SO THE SET OF ORDERED PAIRS {(9, 61), (10, 62), (11, 65), (12, 69), (1, 68), (2, 72), (3, 76)} ARE ANOTHER FORM OF THE DATA IN THE TABLE, WHERE THE FIRST ELEM PAIR IS TIME AND THE SECOND ELEMENT IS THE TEMPERATURE RECORDED AT THAT

Definition 3.14

Given two non-empty sets A and B, the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$ is called the Cartesian product of A and B, denoted by $A \times B$ (read "A cross B").

i.e., $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

NOTE THAT THE SETS A AND B IN THE DEFINITION CAN BE THE SAME OR DIFFERENT.

EXAMPLE 2 IF A = { 1, 2, 3} AND B = { 4, 5}, THEN

$A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

EXAMPLE 3 LET A = { a, b}, THEN FORM $A \times A$.

SOLUTION: $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$.

EXAMPLE 4 LET A = {–1, 0} AND B = {–1, 0, 1}.

FIND $A \times B$ AND ILLUSTRATE IT BY MEANS OF A DIAGRAM.

SOLUTION: $A \times B = \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1)\}$

THE DIAGRAM IS AS SHOWN IN FIGURE 3.25.

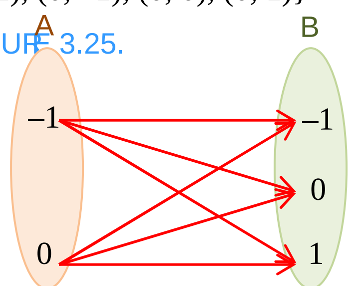
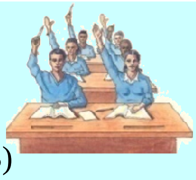


Figure 3.25

Note: $n(A \times B) = n(A) \times n(B)$.

ACTIVITY 3.16



1 LET A = {2, 3} AND B = {0, 1, 2}. FIND:

A $A \times B$ B $B \times A$ C $n(A \times B)$

2 LET A = {a, b}, B = {c, d, e} AND C = {b, c}. FIND:

A $A \times (B \cap C)$ B $A \times (B \cup C)$

C $(A \times B) \cap (A \times C)$ D $(A \times B) \cup (A \times C)$

FROM THE RESULTS OF ACTIVITY 3.16, YOU CONCLUDE THAT:

FOR ANY SETS A, B AND C,

I $A \times B \neq B \times A$, FOR $A \neq B$ *Cartesian product of sets is not commutative.*

II $n(A \times B) = n(A) \times n(B) = n(B \times A)$. *where A and B are finite sets.*

III $A \times (B \cap C) = (A \times B) \cap (A \times C)$. *Cartesian product is distributive over intersection.*

IV $A \times (B \cup C) = (A \times B) \cup (A \times C)$. *Cartesian product is distributive over union.*

Exercise 3.10

1 GIVEN A = {2}, B = {1, 5}, C = {−1, 1}, FIND:

A $A \times B$ B $B \times A$ C $B \times C$ D $A \times (B \cap C)$

E $(A \cup C) \times B$ F $(A \times B) \cup (A \times C)$ G $B \times B$

2 IF $B \times C = \{(1, 1), (1, 2), (1, 3), (4, 1), (4, 2), (4, 3)\}$, FIND:

A B B C C $C \times B$

3 IF $n(A \times B) = 18$ AND $n(A) = 3$, THEN FIND $n(B)$.

4 LET U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} BE THE UNIVERSAL SET, A = {0, 2, 4, 6, 8, 9}, B = {1, 3, 6, 8} AND C = {0, 2, 3, 4, 5}. FIND:

A $A' \times C'$ B $B \times A'$ C $B \times (A' \backslash C)$

5 IF $(2x + 3, 7) = (7, 3y + 1)$, FIND THE VALUES OF x AND y.

3.3.3 Problems Involving Sets

IN THIS SUBSECTION, YOU WILL LEARN HOW TO SOLVE PR INVOLVE SETS, IN PARTICULAR, THE NUMBERS OF ELEMENTS IN S

THE NUMBER OF ELEMENTS THAT ARE IN SET A OR SET DENOTED BY $n(A \cup B)$, MAY NOT NECESSARILY BE $n(A) + n(B)$ AS WE CAN SEE IN THE FIGURE 3.6.

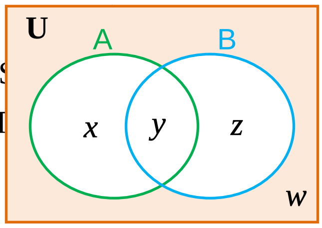


Figure 3.26

IN THIS FIGURE, SUPPOSE THE NUMBER OF ELEMENTS IN THE CLOSED REGIONS OF DIAGRAM ARE DENOTED BY x, y, z AND w.

n (A) = $x + y$ AND n (B) = $y + z$.

SO, n (A) + n (B) = $x + y + y + z$.

n (A ∪ B) = $x + y + z$ = n (A) + n (B) – y

I.E., n (A∪B) = n (A) + n (B) – n (A∩B).

Number of elements in (A∪B)

FOR ANY FINITE SETS A AND B, THE NUMBER OF ELEMENTS THAT ARE IN A ∪ B IS

n (A∪B) = n (A) + n (B) – n (A∩B).

Note: IF A∩B = ∅, THEN n (A∪B) = n (A) + n (B).

EXAMPLE 1 EXPLAIN WHY n (A – B) = n (A) – n (A∩B).

SOLUTION: FROM FIGURE 3.26 ABOVE, n (A) = $x + y$, n (A∩B) = y

n (A) – n (A∩B) = $(x + y) - y = x$,

x IS THE NUMBER OF ELEMENTS IN A THAT ARE NOT IN B. SO, n (A – B) = x.

∴ n (A – B) = x = n (A) – n (A∩B).

FOR ANY FINITE SETS A AND B,

n (A\B) = n (A) – n (A∩B)

EXAMPLE 2 AMONG 1500 STUDENTS IN A SCHOOL, 13 STUDENTS FAILED IN ENGLISH, 12 STUDENTS FAILED IN MATHEMATICS AND 7 STUDENTS FAILED IN BOTH EN MATHEMATICS.

I HOW MANY STUDENTS FAILED IN EITHER ENGLISH OR IN MATHEMATICS?

II HOW MANY STUDENTS PASSED BOTH IN ENGLISH AND IN MATHEMATICS?

SOLUTION: LET E BE THE SET OF STUDENTS WHO FAILED IN ENGLISH, M BE THE SET OF STUDENTS WHO FAILED IN MATHEMATICS AND U BE THE SET OF ALL STUDENTS IN THE S

THEN, n (E) = 13, n (M) = 12, n (E∩M) = 7 AND n (U) = 1500.

I n (E∪M) = n (E) + n (M) – n (E∩M) = 13 + 12 – 7 = 18.

II THE SET OF ALL STUDENTS WHO PASSED IN BOTH SUBJECTS IS U\(E∪M).

n (U\(E∪M)) = n (U) – n (E∪M) = 1500 – 18 = 1482.

Exercise 3.11

1 FOR A = {2, 3, . . . 6} AND B = {6, 7, . . . 10} SHOW THAT:

A $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ B $n(A \times B) = n(A) \times n(B)$

C $n(A \times A) = n(A) \times n(A)$

2 IF $n(C \cap D) = 8$ AND $n(C \backslash D) = 6$ THEN FIND $n(C)$.

3 USING A VENN DIAGRAM, OR A FORMULA, ANSWER EACH OF THE FOLLOWING:

A GIVEN $n(Q \backslash P) = 4$, $n(P \backslash Q) = 5$ AND $n(P) = 7$ FIND $n(Q)$.

B IF $n(R' \cap S') + n(R' \cap S) = 3$, $n(R \cap S) = 4$ AND $n(S' \cap R) = 7$, FIND $n(U)$.

4 INDICATE WHETHER THE STATEMENTS BELOW ARE TRUE OR FALSE FOR ALL FINITE SETS A AND B. IF A STATEMENT IS FALSE GIVE A COUNTER EXAMPLE.

A $n(A \cup B) = n(A) + n(B)$ B $n(A \cap B) = n(A) - n(B)$

C IF $n(A) = n(B)$ THEN A = B D IF A = B THEN $n(A) = n(B)$

E $n(A \times B) = n(A) \cdot n(B)$ F $n(A) + n(B) = n(A \cup B) - n(A \cap B)$

G $n(A' \cup B') = n((A \cup B)')$ H $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$

I $n(A) + n(A') = n(U)$

5 SUPPOSE A AND B ARE SETS SUCH THAT $n(A) = n(B) = 23$ AND $n(A \cap B) = 4$, THEN FIND:

A $n(A \cup B)$ B $n(A \backslash B)$ C $n(A \Delta B)$ D $n(B \backslash A)$

6 IF A = $\{x \mid x$ IS A NON-NEGATIVE INTEGER AND $x^2 = x\}$, THEN HOW MANY PROPER SUBSETS DOES A HAVE?

7 OF 100 STUDENTS, 65 ARE MEMBERS OF A MATHEMATICS CLUB AND 40 ARE MEMBE PHYSICS CLUB. IF 10 ARE MEMBERS OF NEITHER CLUB, THEN HOW MANY STUDENTS MEMBERS OF:

A BOTH CLUBS? B ONLY THE MATHEMATICS CLUB?

C ONLY THE PHYSICS CLUB?

8 THE FOLLOWING VENN DIAGRAM SHOWS AND B. IF $n(A) = 13$, $n(B) = 8$, THEN FIND:

A $n(A \cup B)$ B $n(U)$

C $n(B \backslash A)$ D $n(A \cap B')$

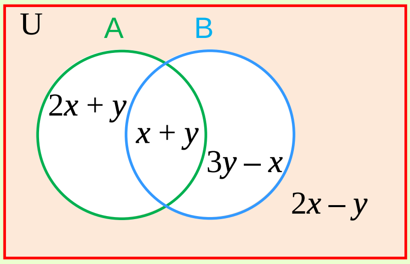


Figure 3.27

Key Terms

complement	infinite set	set
disjoint sets	intersection of sets	subset
element	power set	symmetric difference between sets
empty set	proper subset	union of sets
finite set	relative complement	universal set

Summary

1 A SET IS A WELL-DEFINED COLLECTION OF OBJECTS. THE OBJECTS ITS elements (OR members).

2 SETS ARE DESCRIBED IN THE FOLL:

A VERBAL MET

B LISTING MET

I PARTIAL LISTING II COMPLETE LISTING ME

C SET-BUILDER ME

3 THE universal set IS A SET THAT CONTAINS ALL ELEMENTS CONSIDERA IN A DISCUSSION.

4 THE COMPLEMENT OF A SET A IS THE SET OF ALL ELEMENTS THAT AI SET BUT NOT IN A

5 A SET S IS CALLED finite IF AND ONLY IF IT IS THE EMPTY SET OR HAS EXACTLY n ELEMENTS, WHERE n IS A NATURAL NU. OTHERWISE, IT IS CALLED infinite.

6 A SET A IS A SUBSET OF B IF AND ONLY IF EACH E IN SET B.

7 I P (A), THE POWER SET OF A, IS THE SET OF ALL SUBS

II IF n (A) = n, THEN THE NUMBER OF SUBSE ^{n}.

8 TWO SETS A AND B ARE SA equal IF AND ONLY IF $A \subseteq B$ AND $B \subseteq A$.

9 TWO SETS A AND B ARE SA equivalent IF AND ONLY IF THERE -TO-ONE CORRESPONDENCE B THEIR ELEMENTS.

10 I A SET A IS A proper subset OF SET B, DENOTED BY $A \subset B$, IF AND ONLY $A \subseteq B$ AND $B \not\subseteq A$.

II IF n (A) = n, THEN THE NUMBER OF PROPER SUBS $^n - 1$.

11 OPERATIONS ON SETS; FOR ANY SETS A AND B,

- **I** $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$.
- **II** $A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$.
- **III** $A - B$ (OR $A \backslash B$) $= \{x \mid x \in A \text{ AND } x \notin B\}$.
- **IV** $A \Delta B = \{x \mid x \in (A \cup B) \text{ AND } x \notin (A \cap B)\}$.
- **V** $A \times B = \{(a, b) \mid a \in A \text{ AND } b \in B\}$.

12 PROPERTIES OF UNION, INTERSECTION, SYMMETRIC DIFFERENCE AND CARTESIAN P

FOR ALL SETS A, B AND C:

I COMMUTATIVE PROPERTIES

A $A \cup B = B \cup A$ **B** $A \cap B = B \cap A$ **C** $A \Delta B = B \Delta A$

II ASSOCIATIVE PROPERTIES

A $A \cup (B \cup C) = (A \cup B) \cup C$ **C** $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

B $A \cap (B \cap C) = (A \cap B) \cap C$

III IDENTITY PROPERTIES

A $A \cup \varnothing = A$ **B** $A \cap U = A$ (U is a universal set)

IV DISTRIBUTIVE PROPERTIES

A $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

B $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

C $A \times (B \cup C) = (A \times B) \cup (A \times C)$

D $A \times (B \cap C) = (A \times B) \cap (A \times C)$

V DE MORGAN'S LAW

A $(A \cup B)' = A' \cap B'$ **B** $(A \cap B)' = A' \cup B'$

VI FOR ANY SET A

A $A \cup A' = U$ **B** $(A')' = A$

C $A \cap A' = \varnothing$ **D** $A \times \varnothing = \varnothing$

Review Exercises on Unit 3

1 WHICH OF THE FOLLOWING ARE SETS?

- **A** THE COLLECTION OF ALL TALL STUDENTS IN YOUR CLASS.
- **B** THE COLLECTION OF ALL NATURAL NUMBERS DIVISIBLE BY 3.
- **C** THE COLLECTION OF ALL STUDENTS IN YOUR SCHOOL.
- **D** THE COLLECTION OF ALL INTELLIGENT STUDENTS IN ETHIOPIA.
- **E** THE COLLECTION OF ALL SUBSETS OF THE SET {1, 2, 3, 4, 5}.

2 REWRITE THE FOLLOWING STATEMENTS, USING THE CORRECT NOTATION:

A B IS A SET WHOSE ELEMENTS ARE x, y, z AND w.

B 3 IS NOT AN ELEMENT OF SET B.

C D IS THE SET OF ALL RATIONAL NUMBERS BETWEEN $\sqrt{2}$ AND $\sqrt{5}$.

D H IS THE SET OF ALL POSITIVE MULTIPLES OF 3.

3 WHICH OF THE FOLLOWING PAIRS OF SETS ARE EQUIVALENT?

A $\{1, 2, 3, 4, 5\}$ AND $\{l, m, o, p, q\}$

B $\{x \mid x$ IS A LETTER IN THE WORD MATHEMATICS$\}$ AND $\{y \mid y \in \mathbb{N}, 1 \le y \le 11\}$

C $\{a, b, c, d, e, f, \ldots\ m\}$ AND $\{1, 2, 3, 4, 5, \ldots 13\}$

4 WHICH OF THE FOLLOWING REPRESENT EQUAL SETS?

A = $\{a, b, c, d\}$ B = $\{x, y, z, w\}$

C = $\{x \mid x$ IS ONE OF THE FIRST FOUR LETTERS IN THE ENGLISH ALPHABET$\}$

D = $\varnothing$ E = $\{0\}$ F = $\{x \mid x \neq x\}$ G = $\{x \in \mathbb{Z} \mid -1 < x < 1\}$

5 IF U = $\{a, b, c, d, e, f, g, h\}$, A = $\{b, d, f, h\}$ AND B = $\{a, b, e, f, g, h\}$, FIND THE FOLLOWING:

A A' **B** B' **C** A∩B **D** (A ∩ B)' **E** A' ∩ B'

6 IN THE VENN DIAGRAM GIVEN BELOW, WRITE THE REGION LABELLED BY I, II, III AND IV IN TERMS OF A AND B.

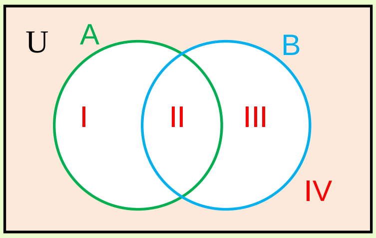


Figure 3.28

7 FOR EACH OF QUESTIONS A, B AND C, COPY THE FOLL… VENN DIAGRAM AND SHADE THE REGIONS TH…

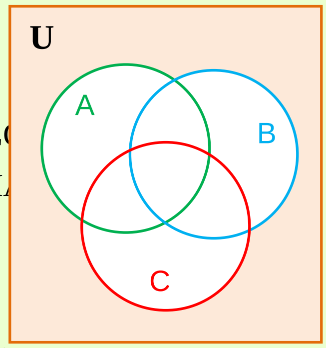


Figure 3.29

A A ∩ (B ∩ C). **B** A\(B ∩ C).

C A ∪ (B\C).

8 LET U = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, A = $\{1, 2, 3, 4, 5\}$,

B = $\{0, 2, 1, 6, 8\}$ AND C = $\{3, 6, 9\}$. THEN FIND:

A A' **B** B\A **C** A ∩ C'

D C × (A∩B) **E** (B\A)×C

9 SUPPOSE B IS A PROPER SUBSET OF C,

- **A** IF n (C) = 8, WHAT IS THE MAXIMUM NUMBER OF ELEMENTS IN B?
- **B** WHAT IS THE LEAST POSSIBLE NUMBER OF ELEMENTS IN B?

10 IF n (U) = 16, n (A) = 7 AND n (B) = 12, FIND:

- **A** n (A') **B** n (B')
- **C** GREATEST n (A $\cap$ B) **D** LEAST n (A $\cup$ B)

11 IN A CLASS OF 31 STUDENTS, 22 STUDENTS STUDY PHYSICS, 20 STUDENTS STUDY CH AND 5 STUDENTS STUDY NEITHER. CALCULATE THE NUMBER OF STUDENTS WHO ST SUBJECTS.

12 SUPPOSE A AND B ARE SETS SUCH THAT A $\cup$ B HAS 20 ELEMENTS, A $\cap$ B HAS 7 ELEMENTS, AND THE NUMBER OF ELEMENTS IN B IS TWICE THAT OF A. WHAT IS THE NUMBER O ELEMENTS IN:

- **A** A? **B** B?

13 STATE WHETHER EACH OF THE FOLLOWING IS: infinite finite

- **A** {x | x IS AN INTEGER LESS THAN 5}
- **B** {x | x IS A RATIONAL NUMBER BETWEEN 0 AND 1}
- **C** {x | x IS THE NUMBER OF POINTS ON A 1 CM-LONG LINE SEGMENT}
- **D** THE SET OF TREES FOUND IN ADDIS ABABA.
- **E** THE SET OF “TEFF” IN 1,000 QUINTALS.
- **F** THE SET OF STUDENTS IN THIS CLASS WHO ARE 10 YEARS OLD.

14 HOW MANY LETTERS IN THE ENGLISH ALPHABET PRECEDE THE LETTER Q? (THINK OF A SHORTCUT METHOD).

15 OF 100 STAFF MEMBERS OF A SCHOOL, 48 DRINK COFFEE, 25 DRINK BOTH TEA AND C AND EVERYONE DRINKS EITHER COFFEE OR TEA. HOW MANY STAFF MEMBERS DRIN

16 GIVEN THAT SET A HAS 15 ELEMENTS AND SET B HAS 12 ELEMENTS, DETERMINE EA FOLLOWING:

- **A** THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN A $\cup$ B
- **B** THE MINIMUM POSSIBLE NUMBER OF ELEMENTS IN A $\cup$ B
- **C** THE MAXIMUM POSSIBLE NUMBER OF ELEMENTS IN A $\cap$ B
- **D** THE MINIMUM POSSIBLE NUMBER OF ELEMENTS IN A $\cap$ B